basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 24pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

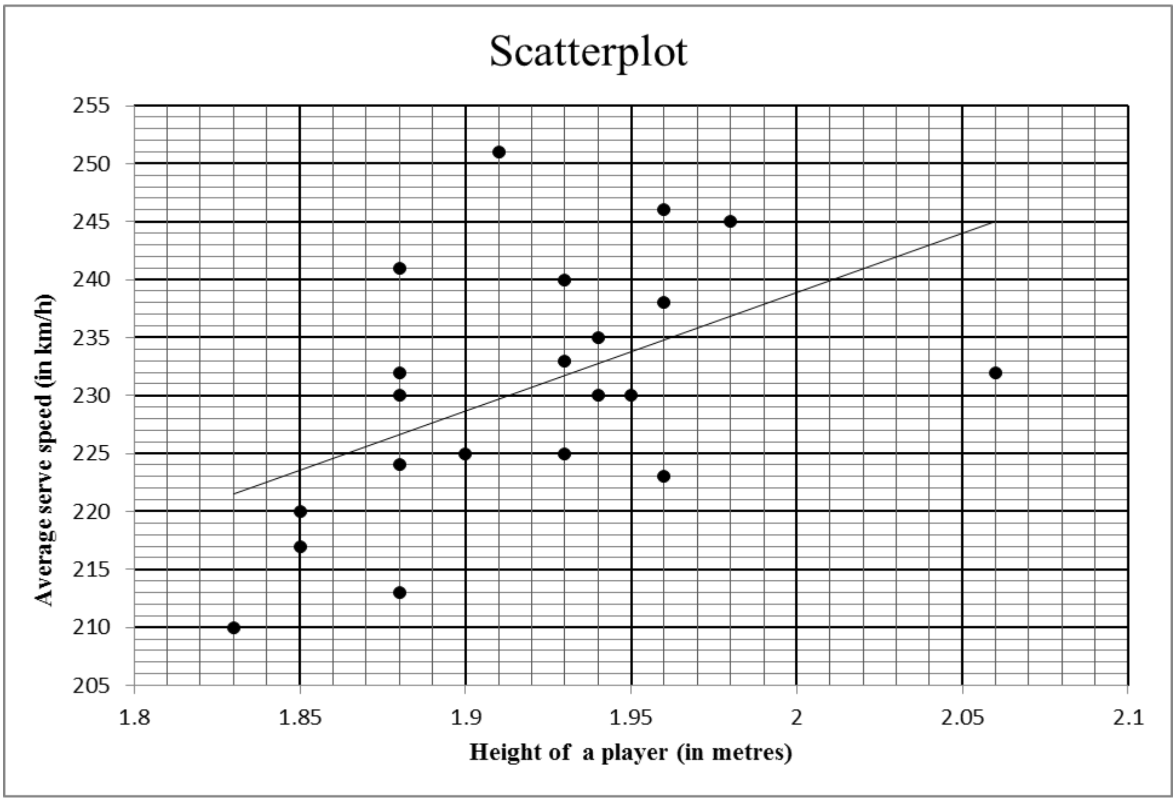
QUESTION/*VRAAG* 1

1.1.1	140 items	✓ answer (1)
1.1.2	Modal class/*modale klas:* $20 < x \leq 30$ minutes **OR/*OF*** $20 \leq x < 30$ minutes	✓ answer (1) ✓ answer (1)
1.1.3	Number of minutes taken = 20 minutes	✓ answer (1)
1.1.4	140 – 126 [Accept: 124 to 128] 14 orders (12 to 16) Answer only: Full marks	✓ 126 ✓ answer (2)
1.1.5	75th percentile is at 105 items =37 minutes [accept 36 – 38 minutes] Answer only: Full marks	✓ 105 ✓ answer (2)
1.1.6	Lower quartile is at 35 items =21,5 min [accept 21 – 23 min] IQR = 37 – 21,5 = 15,5 min [accept 13 – 17 min] Answer only: Full marks	✓ lower quartile (Q_1) ✓ answer (2)

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1(a)	$\bar{x} = \frac{1420}{15}$ $= R94{,}666.. = R94{,}67$ Answer only: Full marks	✓ 1420 ✓ answer (2)
1.2.1(b)	$\sigma = R22{,}691... = R22{,}69$	✓✓ answer (2)
1.2.2(a)	They both collected the **same (equal) amount** in tips, i.e. R1 420 over the 15-day period. *Hulle albei het **dieselfde bedrag** met fooitjies ontvang, nl. R1 420 oor die 15 dae-tydperk*	✓ answer (1)
1.2.2(b)	Mary's standard deviation is smaller than Reggie's which suggests that there was **greater variation in the amount of tips that Reggie collected** each day compared to the number of tips that Mary collected each day. *Marie se standaardafwyking is kleiner as Reggie s'n wat beteken dat daar **groter variasie/verspreiding in die fooitjies was wat Reggie** elke dag **ontvang** het in vergelyking met die getal fooitjies wat Marie elke dag ontvang het.*	✓ explanation (1)
		[15]

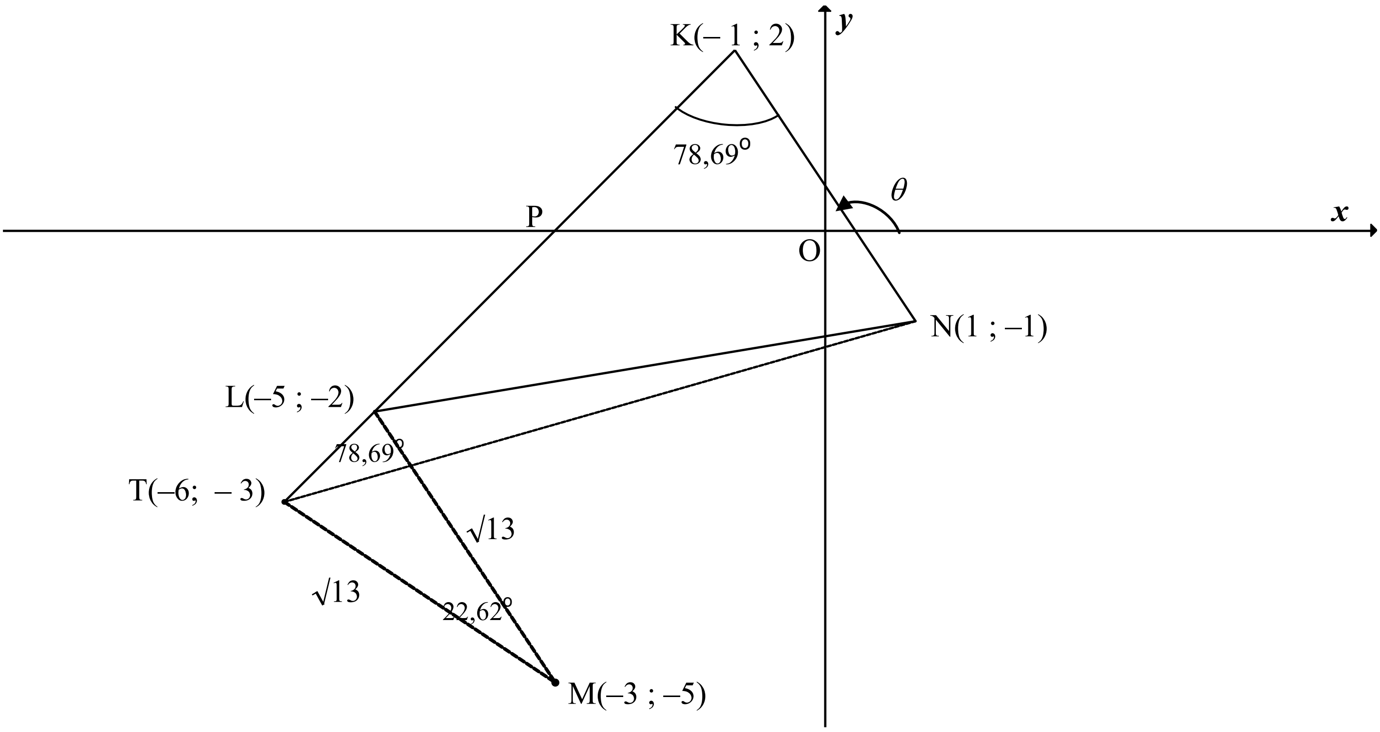
QUESTION/*VRAAG* 2



2.1	251 km/h	✓ answer (1)
2.2.1	$r = 0{,}52$ **OR** C	✓ answer (1)
2.2.2	The points are **fairly scattered** and the least squares regression line is increasing. *Die punte is **redelik verspreid** en die kleinstekwadrate-regressielyn neem toe.*	✓ reason (1)
2.3	There is a weak positive relation hence the height could have an influence *Daar is 'n swak positiewe verband, tog kan die lengte 'n invloed hê.* **OR/*OF*** There is no conclusive evidence that the height of a player will influence his/her tennis serve speed. *Daar is geen duidelike bewys dat die lengte van die speler sy/haar afslaanspoed kan beïnvloed nie.* **OR/*OF*** There is no conclusive evidence that a taller person will serve faster than a shorter person. *Daar is geen duidelike bewys dat 'n langer speler vinniger sal afslaan as 'n korter een nie.*	✓ answer (1) ✓ answer (1) ✓ answer (1)

2.4	For (0 ; 27,07), it means that the player has a height of 0 m but can serve at a speed of 27,07 km/h. **It is impossible for a person to have a height of 0 m**. *(0 ; 27,07) beteken dat 'n speler 'n lengte van 0* m *kan hê en teen 'n spoed van 27,07* km/h *kan afslaan.* ***Dit is onmoontlik om 'n lengte van 0*** **m** ***te hê.*** **OR/OF** This means that the **player does not exist and therefore cannot serve and have a serve speed.** *Dit beteken dat die* ***speler nie bestaan nie en daarom nie kan afslaan en 'n afslaanspoed hê nie.***	✓ explanation (1) ✓ explanation (1)
		[5]

QUESTION/*VRAAG* 3



3.1.1	$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2-(-1)}{-1-1}$ $= -\frac{3}{2}$ Answer only: Full marks	✓ correct substitution ✓ answer (2)
3.1.2	$\tan\theta = m_{KN} = -\frac{3}{2}$ $\theta = 180° - 56{,}31°$ $\theta = 123{,}69°$ Answer only: Full marks	✓ $\tan\theta = m_{KN} = -\frac{3}{2}$ ✓ answer (2)
3.2	Inclination KL = 123,69° – 78,69° = 45° [ext ∠Δ] $\tan 45° = m_{KL} = 1$	✓ S ✓ $\tan 45° = m_{KL} = 1$ (2)
3.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y = x + 3$	✓ substitute (– 1 ; 2) and m ✓ equation (2) ✓ substitute (– 1 ; 2) and m ✓ equation (2)

3.4	$KN=\sqrt{(1+1)^2+(-1-2)^2}$ $KN=\sqrt{13}$ or 3,61 Answer only: Full marks	✓ substitute K and N into distance formula ✓ answer (2)
3.5.1	$(x+3)^2+(y+5)^2=13$ ...(1) L is a point on KL $y=x+3$ …(2) (2) in (1): $(x+3)^2+(x+3+5)^2=13$ $x^2+6x+9+x^2+16x+64=13$ $2x^2+22x+60=0$ $x^2+11x+30=0$ $(x+5)(x+6)=0$ $x=-5$ or $x=-6$ $y=-2$ or $y=-3$ L(–5 ; –2) or (–6 ; –3) **OR/OF** $(x+3)^2+(y+5)^2=13$ ...(1) L is a point on KL $y=x+3$ $\therefore x=y-3$ …(2) (2) in (1): $(y-3+3)^2+(y+5)^2=13$ $y^2+y^2+10y+25=13$ $2y^2+10y+12=0$ $y^2+5y+6=0$ $(y+2)(y+3)=0$ $y=-2$ or $y=-3$ $x=-5$ or $x=-6$ L(–5 ; –2) or (–6 ; –3)	✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ x-values ✓ y-values (5) ✓ equation (1) ✓ substituting eq (2) ✓ standard form ✓ y-values (both) ✓ x-values (both) (5)
3.5.2	Midpoint of KM: (– 2 ; – 1,5) $\therefore \frac{x_L+1}{2}=-2$ and $\frac{y_L-1}{2}=-\frac{3}{2}$ $\therefore$ L(– 5 ; – 2) **OR/*OF*** $m_{KN}=m_{LM}$ $\frac{y-(-5)}{x-(-3)}=-\frac{3}{2}$ $2(x+3+5)=-3(x+3)$ $2x+16=-3x-9$ $5x=-25$ $x=-5$ $\therefore$ L(– 5 ; – 2) Answer only: Full marks	✓ midpoint of KM ✓x value ✓ y value (3) ✓ $m_{LM}=m_{KN}$ ✓x value ✓ y value (3)

	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ $\therefore$ L(– 1 – 4 ; 2 – 4) $\therefore$ L(– 5 ; – 2) **OR/OF** N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ $\therefore$ L(– 3 – 2 ; – 5 + 3) $\therefore$ L(– 5 ; – 2)	
	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ $\therefore$ L(– 1 – 4 ; 2 – 4) $\therefore$ L(– 5 ; – 2) **OR/OF** N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ $\therefore$ L(– 3 – 2 ; – 5 + 3) $\therefore$ L(– 5 ; – 2)	
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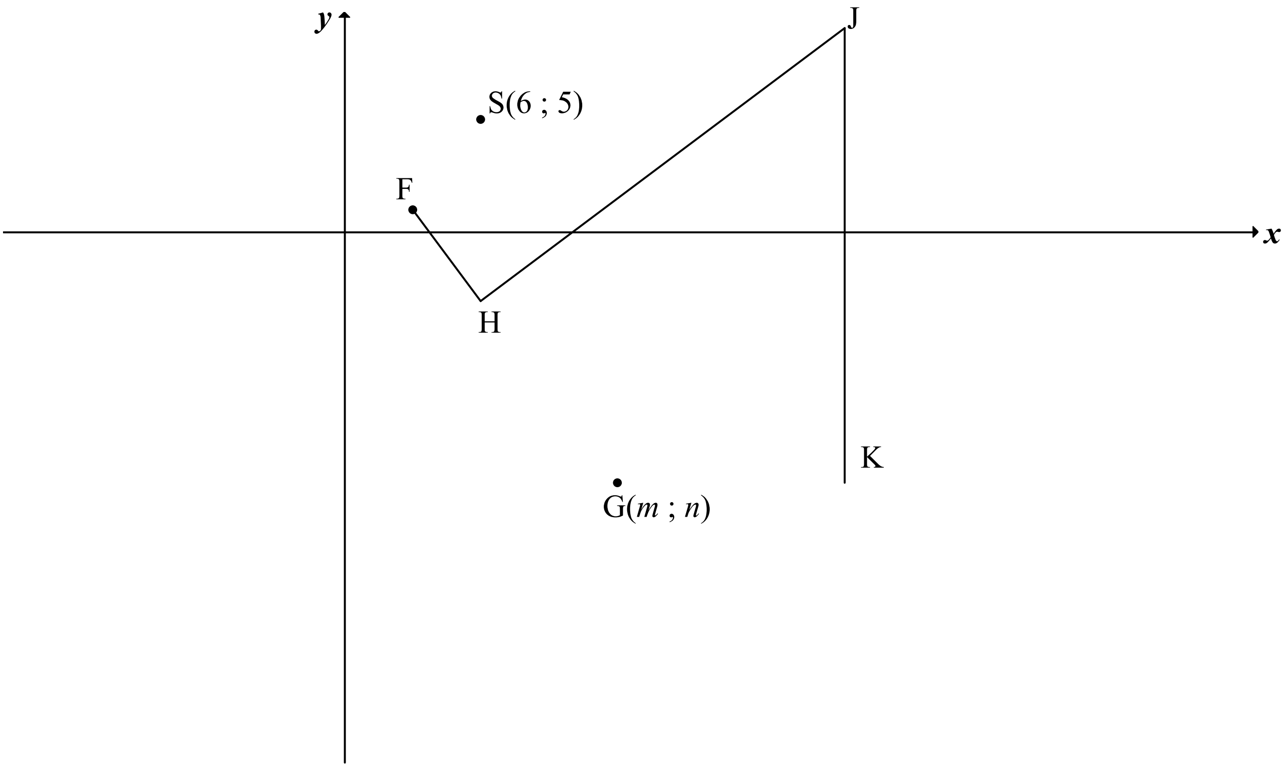
	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ $\therefore$ L(– 1 – 4 ; 2 – 4) $\therefore$ L(– 5 ; – 2) **OR/OF** N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ $\therefore$ L(– 3 – 2 ; – 5 + 3) $\therefore$ L(– 5 ; – 2)	✓ transformation ✓x value ✓ y value (3)
3.6	T(–6 ; –3) (from Question 3.5.1) $\text{KT} = \sqrt{(-1-(-6))^2 + (2-(-3))^2}$ $= \sqrt{50}$ $\text{KN} = \sqrt{13}$ (CA from 3.4) Area of ΔKTN $= \frac{1}{2}\text{KT.KN}\sin\text{L}\hat{\text{K}}\text{N}$ $= \frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$ = 12,50 square units	✓ coordinates of T ✓ length of KT ✓ substitution into area rule ✓ answer (4)

	OR/*OF* In ΔKLM: $\frac{TL}{\sin 22,62°} = \frac{\sqrt{13}}{\sin 78,69°}$ $TL = 1,414..$ $KL = \sqrt{(-1-(-5))^2 + (2-(-2))^2}$ $= \sqrt{32}$ $\therefore$ KT = 7,0708… Area of $\Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$ $= \frac{1}{2}(7,0708).\sqrt{13} \sin 78,69°$ = 12,50 square units	✓ length of TL ✓ length of KT ✓ substitution into area rule ✓ answer (4)
		[22]

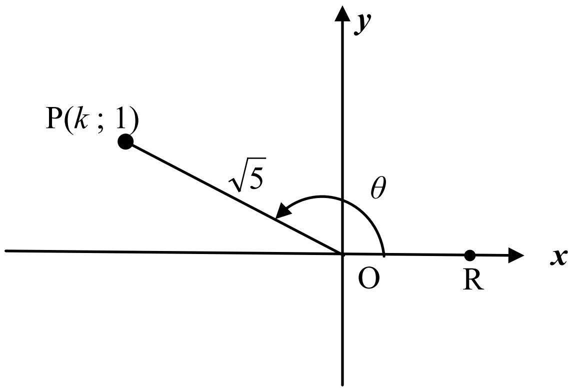
QUESTION/*VRAAG* 4



4.1	$F(3;1)$	✓x value ✓ y value (2)
4.2	$FS = \sqrt{(6-3)^2 + (5-1)^2}$ $FS = 5$	✓ substitution of F & S ✓ answer (2)
4.3	FH(FS) : HG = 1 : 2 $\therefore$ HG = 2 FH = 10	✓ HG = 10 (1)
4.4	Tangents from common/same point / *Raaklyne vanaf gemeenskaplike of dieselfde punt*	✓ answer (1)
4.5.1	$F\hat{H}J = 90°$ [tan ⊥ radius / *rkl ⊥ radius*] $FJ^2 = 20^2 + 5^2$ [Pyth theorem/*stelling*] $FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62	✓ S ✓ R ✓ S ✓ answer (4)
4.5.2	$(x-m)^2 + (y-n)^2 = 100$	✓ answer (1)

4.5.3	K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ F, H and G are collinear [HJ is a common tangent] *F, H en G is saamlynig [HJ is 'n gemeemskaplike raaklyn]* $FG^2 = (12-3)^2 + (n-1)^2$ $15^2 = 81 + (n-1)^2$ $(n-1)^2 = 144$ $n - 1 = \pm 12$ **OR/*OF*** $\boxed{n^2 - 2n - 143 = 0 \quad (n+11)(n-13) = 0 \quad n = -11 \text{ or } n \neq 13}$ $n \neq 13$ or $n = -11$ $\therefore$ G(12; –11) **OR/*OF*** K(22; n) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] $m = 22 - 10$ $m = 12$ Let J(22 ; y): $FJ^2 = (22-3)^2 + (y-1)^2$ $425 = 361 + y^2 - 2y + 1$ $0 = y^2 - 2y - 63$ $0 = (y-9)(y+7)$ $\therefore y = 9$ or/of $y \neq -7$ $\therefore n = 9 - 20 = -11$ $\therefore$ G(12; –11)	✓ K(22; n) ✓ value of m ✓ subst. of F and G in distance formula ✓ FG = 15 ✓ simplification/ standard form ✓ value of n ✓ coordinates of G (7) ✓ K(22; n) ✓ value of m ✓ subst. of F and J in distance formula ✓ FJ = $\sqrt{425}$ ✓ standard form ✓ value of n ✓ coordinates of G (7)
		[18]

QUESTION/*VRAAG* 5

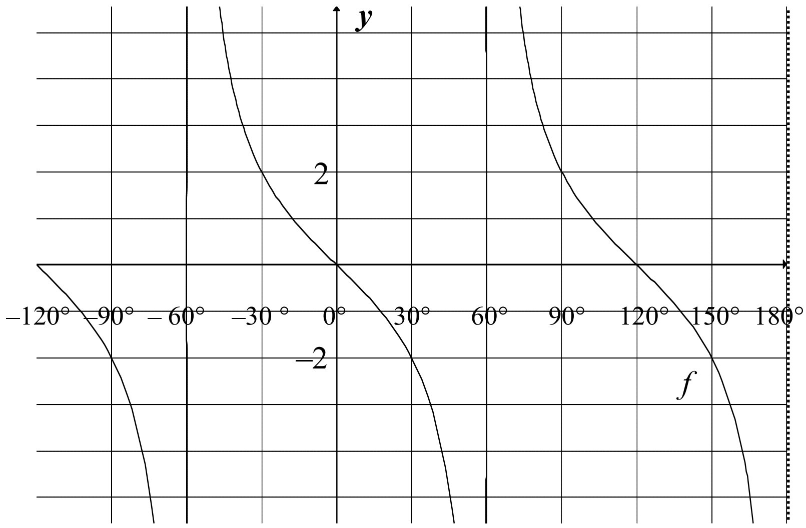


5.1.1	$k^2 = (\sqrt{5})^2 - 1^2$ $= 4$ $k = -2$ Answer only: full marks	✓ substitution into theorem of Pythagoras ✓ answer (2)
5.1.2(a)	$\tan\theta = -\frac{1}{2}$	✓answer (1)
5.1.2(b)	$\cos(180° + \theta) = -\cos\theta$ $= \frac{2}{\sqrt{5}}$ Answer only: full marks	✓ reduction ✓ answer (2)
5.1.2(c)	$\sin(\theta + 60°) = \frac{a+b}{\sqrt{20}}$ LHS= $\sin\theta\cos 60° + \cos\theta\sin 60°$ $= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{2}\right) + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1-2\sqrt{3}}{2\sqrt{5}}$ $= \frac{1-2\sqrt{3}}{\sqrt{20}}$	✓ expansion ✓ subst of sin θ ✓ subst of cos θ ✓ both special ∠s ✓ $\frac{1-2\sqrt{3}}{2\sqrt{5}}$ (5)
5.1.3	$\tan\theta = -\frac{1}{2}$ $\therefore \theta = 180° - 26{,}57°$ $\therefore \theta = 153{,}43°$ $\tan(2\theta - 40°) = \tan[(2 \times 153{,}43°) - 40°]$ $= \tan 266{,}87°$ $=18{,}3$	✓ θ ✓ substitution ✓ answer (3)

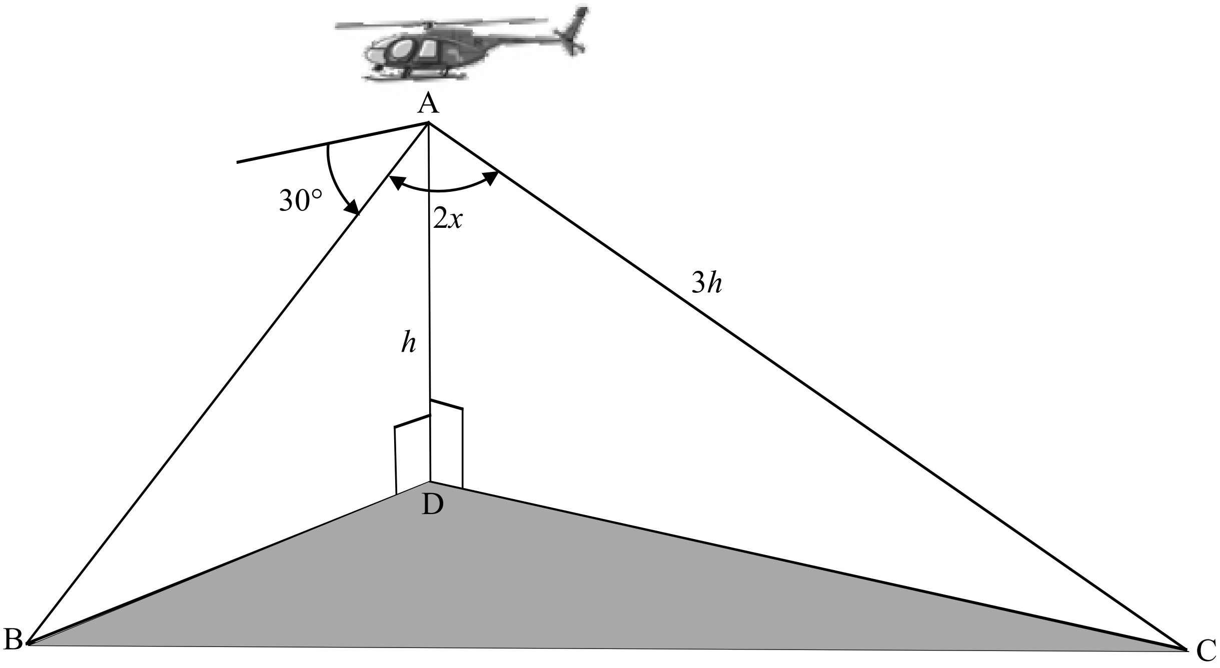
5.2	LHS $= \dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x}$ RHS $= 2\tan 2x$ $= \dfrac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \dfrac{\cos^2 x + 2\sin x\cos x + \sin^2 x - \cos^2 x + 2\sin x\cos x - \sin^2 x}{\cos^2 x - \sin^2 x}$ $= \dfrac{2(2\sin x\cos x)}{\cos^2 x - \sin^2 x}$ $= \dfrac{2\sin 2x}{\cos 2x}$ $= 2\tan 2x$ $=$ RHS	✓ single fraction ✓ expansion ✓ simplification (both) ✓ double ∠ identity ✓ double ∠ identity (5)
	OR/*OF* LHS $= \dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x}$ RHS $= 2\tan 2x$ $= \dfrac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \dfrac{(\cos x + \sin x + \cos x - \sin x)(\cos x + \sin x - \cos x + \sin x)}{\cos^2 x - \sin^2 x}$ $= \dfrac{(2\cos x)(2\sin x)}{\cos^2 x - \sin^2 x}$ $= \dfrac{2(2\sin x\cos x)}{\cos^2 x - \sin^2 x}$ $= \dfrac{2\sin 2x}{\cos 2x}$ $= 2\tan 2x$ $=$ RHS	✓ single fraction ✓ difference of two squares ✓ simplification (both) ✓ double ∠ identity ✓ double ∠ identity (5)
	OR/*OF* RHS $= 2\tan 2x$ $= \dfrac{2\sin 2x}{\cos 2x}$ $= \dfrac{2(2\sin x.\cos x)}{\cos^2 x - \sin^2 x}$ $= \dfrac{4\sin x.\cos x}{\cos^2 x - \sin^2 x}$ $= \dfrac{1 + 2\sin x.\cos x - (1 - 2\sin x.\cos x)}{\cos^2 x - \sin^2 x}$ $= \dfrac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$ $= \dfrac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)} - \dfrac{(\cos x - \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$ $= \dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} =$ LHS	✓ double ∠ identity ✓ double ∠ identity ✓ identity & method ✓ factorising numerator and denominator ✓ writing as 2 terms (5)

5.3	$\sum_{A=38^\circ}^{52^\circ} \cos^2 A$ $= \cos^2 38° + \cos^2 39° + \cos^2 40° + ... + \cos^2 51° + \cos^2 52°$ $= \sin^2 52° + \sin^2 51° + \sin^2 50° + ... + \cos^2 51° + \cos^2 52°$ $= 7(1) + \cos^2 45°$ $= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$ or $= 7 + \left(\frac{1}{\sqrt{2}}\right)^2$ $= 7\frac{1}{2}$	✓ expansion ✓ co ratio ✓ $\cos^2 45°$ ✓ 7 × identity ✓ answer (5)
	OR/*OF*	
	$\sum_{A=38^\circ}^{52^\circ} \cos^2 A$ $= \cos^2 38° + \cos^2 39° + \cos^2 40° + ... + \cos^2 51° + \cos^2 52°$ $= (\cos^2 38° + \sin^2 52°) + (\cos^2 39° + \sin^2 51°)... + \cos^2 45°$ $= 7(1) + \cos^2 45°$ $= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$ or $= 7 + \left(\frac{1}{\sqrt{2}}\right)^2$ $= 7\frac{1}{2}$	✓ expansion ✓ pairing ✓ $\cos^2 45°$ ✓ 7 × identity ✓ answer (5)
		[23]

QUESTION/*VRAAG* 6

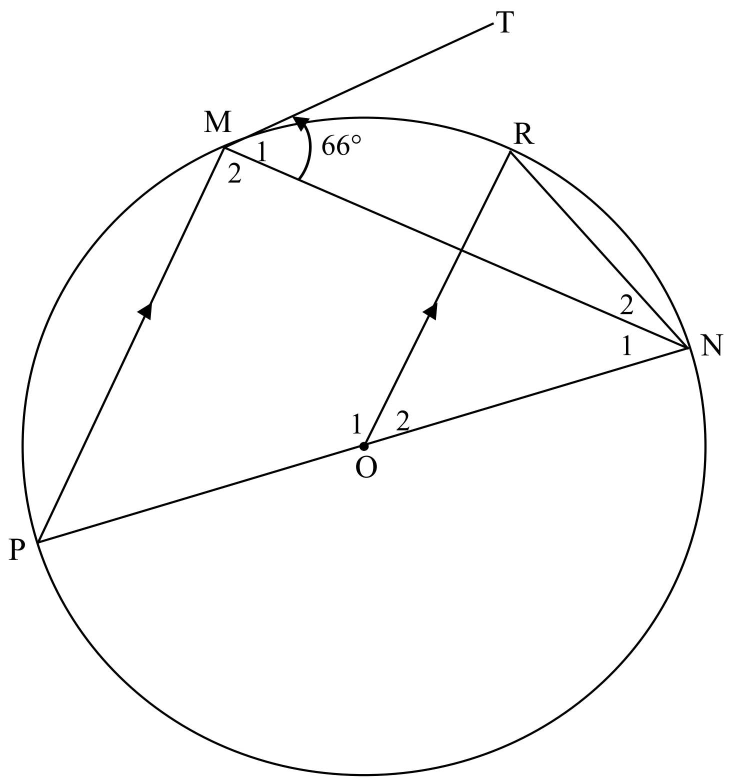
6.1	Period = 120°	✓ answer (1)
6.2	$2 = -2\tan\frac{3}{2}x$ $\tan\left(\frac{3}{2}t\right) = -1$ $\frac{3}{2}t = 135° + k.180°$ **OR/*OF*** $\frac{3}{2}t = -45° + k.180°$ $t = 90° + k.120°\ ;\ k \in Z$ $t = -30° + k.120°\ ;\ k \in Z$ **OR/*OF*** $2 = -2\tan\frac{3}{2}x$ $\tan\left(\frac{3}{2}t\right) = -1$ $\frac{3}{2}t = 135° + k.360°$ **o**r/of $\frac{3}{2}t = 315° + k.360°$ $t = 90° + k.240°$ **o**r.of $t = 210° + k.240°\ ;\ k \in Z$	✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3) ✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3)
6.3		✓asymptotes: $x = \pm 60°$; $x = 180°$ ✓x-intercepts 0°; ±120° ✓ negative shape ✓ (90° ; 2) or (− 30° ; 2) or (30° ; −2) or (−90° ; −2) (4)
6.4	$x \in (-60°\ ;\ -30°]$ or $(60°\ ;\ 90°]$ **OR/*OF*** $-60° < x \leq -30°$ or $60° < x \leq 90°$	✓ interval ✓ interval ✓ notation (3) ✓ interval ✓ interval ✓ notation (3)
6.5	$g(x) = -2\tan\left[\frac{3}{2}(x + 40°)\right] = f(x + 40°)$ Translation of 40° to the left / skuif met 40° links	✓ Translation of 40° ✓ to the left (2)
		[13]

QUESTION/*VRAAG* 7



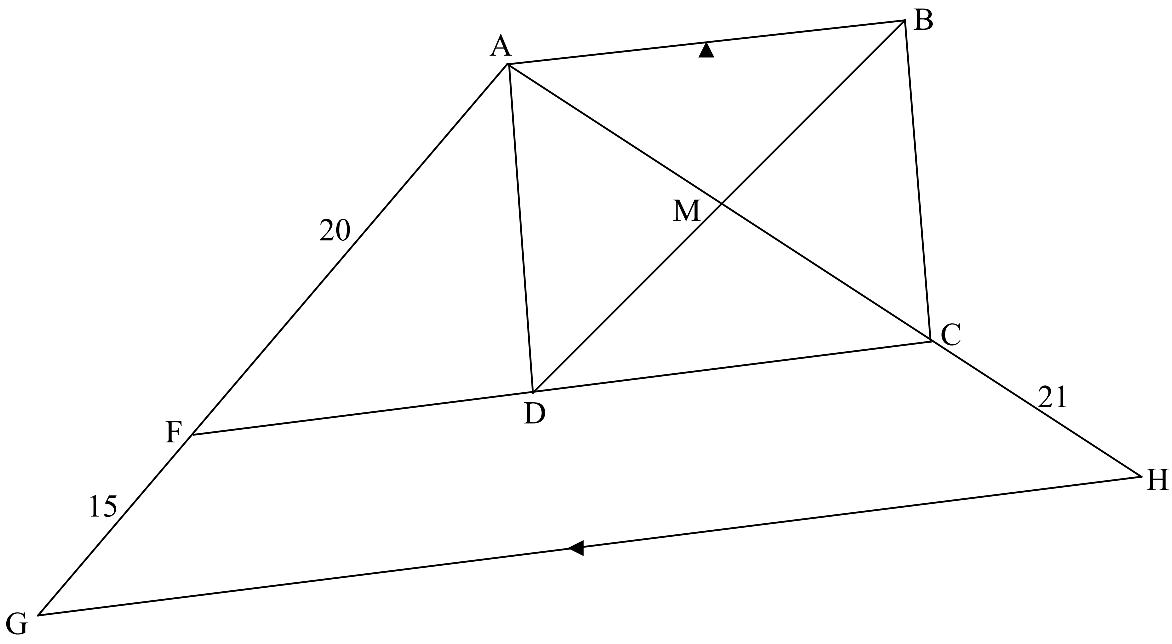
7.1	$\hat{ABD} = 30°$ $\sin 30° = \frac{h}{AB}$ $AB = \frac{h}{\sin 30°}$ **OR** $AB = \frac{h}{\frac{1}{2}}$ **OR** $AB = 2h$ **OR/*OF*** $\hat{BAD} = 60°$ $\cos 60° = \frac{h}{AB}$ $AB = \frac{h}{\cos 60°}$ **OR** $AB = \frac{h}{\frac{1}{2}}$ **OR** $AB = 2h$	✓ $\hat{ABD} = 30°$ ✓ answer (2) ✓ $\hat{BAD} = 60°$ ✓ answer (2)
7.2	$BC^2 = AB^2 + AC^2 - 2AB.AC\cos B\hat{A}C$ $= (2h)^2 + (3h)^2 - 2(2h)(3h)\cos 2x$ $= 13h^2 - 12h^2(2\cos^2 x - 1)$ $= 13h^2 - 24h^2\cos^2 x + 12h^2$ $= 25h^2 - 24h^2\cos^2 x$ $BC = h\sqrt{25 - 24\cos^2 x}$	✓ use of cosine rule in ΔABC ✓ substitution ✓ double angle identity ✓ $25h^2 - 24h^2\cos^2 x$ (4)
		[6]

QUESTION/*VRAAG* 8



8.1.1	$\hat{P} = \hat{M}_1 = 66°$ [tan chord theorem/*raaklyn koordst*]	✓S ✓R (2)
8.1.2	$\hat{M}_2 = 90°$ [$\angle$ in semi circle/$\angle$ *in halfsirkel*]	✓S ✓R (2)
8.1.3	$\hat{N}_1 = 180° - (90° + 66°)$ [sum of $\angle$s of /*som van* $\angle$*e* ΔMNP] $= 24°$	✓S (1)
8.1.4	$\hat{O}_2 = \hat{P} = 66°$ [corres. $\angle$s;/*ooreenk* $\angle$*e,* PM ‖ OR]	✓S ✓R (2)
8.1.5	$\hat{R} + \hat{N}_1 + \hat{N}_2 = 180° - 66°$ [sum of $\angle$s of/*som van* $\angle$*e* ΔRNO] $= 114°$ $\hat{R} = \hat{N}_1 + \hat{N}_2 = 57°$ [$\angle$s opposite = radii/$\angle$*e teenoor = radii*] $\therefore\ \hat{N}_2 = 33°$ **OR/*OF*** $P\hat{O}R = 114°$ [$\angle$s on straight line/$\angle$*e op reguitlyn*] $P\hat{N}R = 57°$ [$\angle$ at centre = twice $\angle$ at circumference/ *midpts*$\angle$ *= 2 × omtreks*$\angle$] $\therefore\ \hat{N}_2 = 33°$	✓S ✓S/R ✓S (3) ✓S ✓S/R ✓S (3)

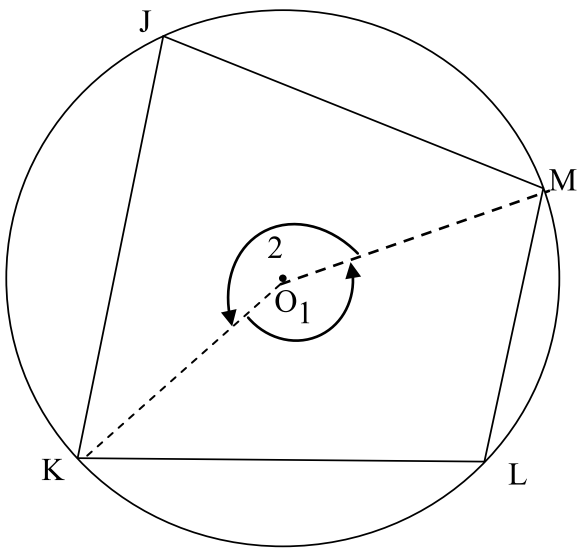
8.2



8.2.1	FC ‖ AB ‖ GH [opp sides of **rectangle**/*teenoorst sye v reghoek*]	✓ R (1)
8.2.2	$\frac{AC}{CH} = \frac{AF}{FG}$ [line ‖ one side of Δ] **OR** [prop theorem; FC ‖ GH] *[lyn ‖ een sy van Δ]* ***OF*** *[eweredighst; FC ‖ GH]* $\frac{AC}{21} = \frac{20}{15}$ $AC = \frac{20 \times 21}{15}$ $= 28$ DB = AC = 28 [diags of rectangle =/*hoeklyne v reghoek* =] $DM = \frac{1}{2}DB = 14$ [diags of rectangle bisect/*hoekl v reghoek halveer*]	✓S ✓R ✓ AC ✓ S ✓ S (5)
		[16]

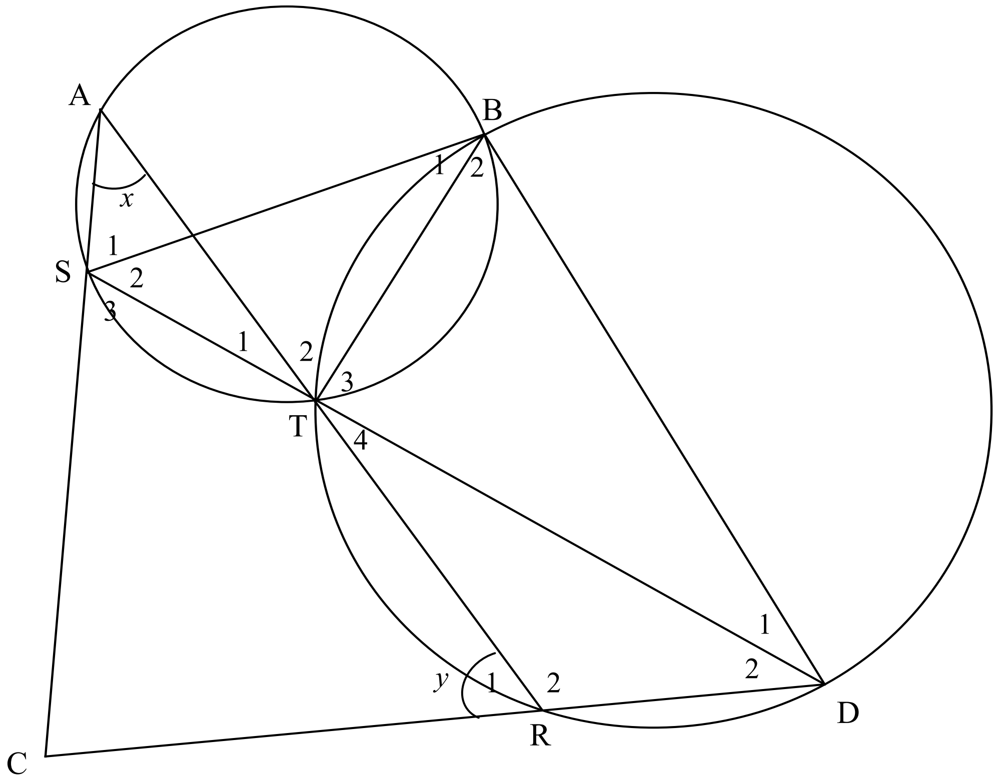
QUESTION/*VRAAG* 9

9.1



9.1	Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO* Proof: $\hat{O}_1 = 2\hat{J}$ [∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]* $\hat{O}_2 = 2\hat{L}$ [∠ at centre = twice ∠ at circumference] $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s around a point /∠*e om 'n punt*] $\therefore 2\hat{J} + 2\hat{L} = 360°$ $\therefore 2(\hat{J} + \hat{L}) = 360°$ $\therefore \hat{J} + \hat{L} = 180°$	✓ construction ✓ S/R ✓ S ✓ S/R ✓ S (5)
	OR/*OF*	
	Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO* Proof: Let $\hat{J} = x$ $\hat{O}_1 = 2x$ [∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]* $\hat{O}_2 = 360° - 2x$ [∠s around a point /∠*e om 'n punt*] $\therefore \hat{L} = 180° - x$ [∠ at centre = twice ∠ at circumference] $\therefore \hat{J} + \hat{L} = 180°$	✓ construction ✓ S ✓ R ✓ S/R ✓ S (5)

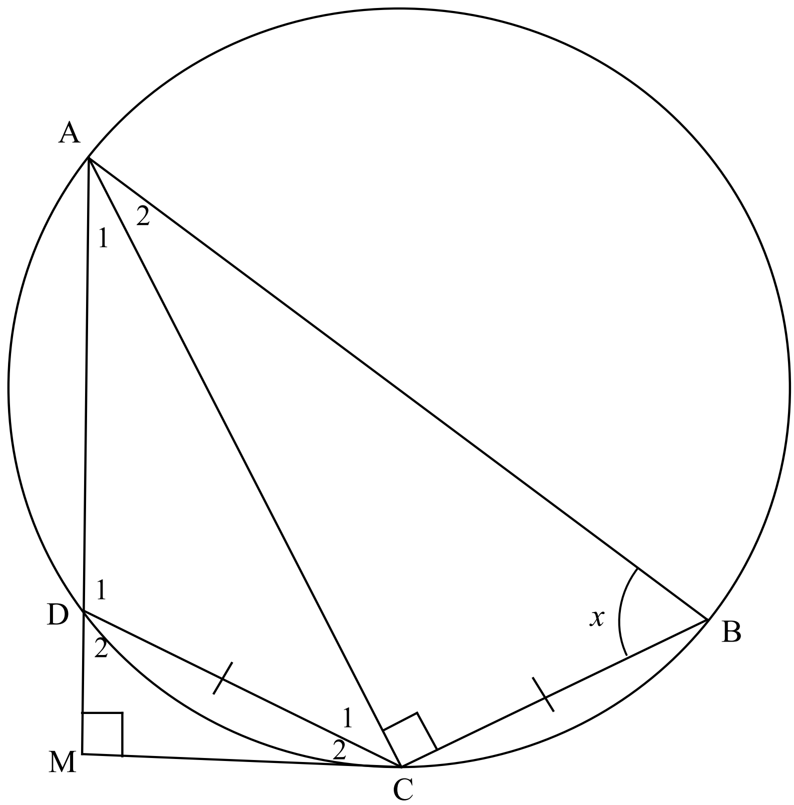
9.2



9.2.1(a)	$\hat{B}_1 = x$ [∠s in same seg/*∠e in dieselfde segm*]	✓ S ✓ R (2)
9.2.1(b)	$\hat{B}_2 = y$ [ext ∠ of cyclic quad/*buite∠ koordevh*]	✓ S ✓ R (2)
9.2.2	$\hat{C} = 180° - (x + y)$ [sum of ∠s of/*som v ∠e*, ΔACR] $S\hat{B}D + \hat{C} = x + y + 180° - (x + y)$ $S\hat{B}D + \hat{C} = 180°$ SCDB is a cyclic quad [converse opp angles of cyclic quad] [*omgekeerde teenoorst ∠e koordevh*] **OR/*OF*** $\hat{S}_1 = \hat{T}_2$ [∠s in same segment/*∠e in dies. segment*] $\hat{T}_2 = \hat{D}_1 + \hat{D}_2 = B\hat{D}R$ [ext ∠ of cyc quad/*buite∠ koordevh*] $\therefore\ \hat{S}_1 = B\hat{D}R$ $\therefore$ SCDB is cyc quad [ext ∠ of quad = opp ∠/*buite∠ = tos ∠*]	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3)

9.2.3	$\hat{T}_4 = y - 30°$ [ext $\angle$ of/*buite* $\angle$ ΔTDR] $\hat{T}_1 = y - 30°$ [vert opp $\angle$s =/*regoorst* $\angle$*e* =] $y - 30° + x + 100° = 180°$ [sum of $\angle$s of/*som v* $\angle$*e*, ΔAST] $\therefore x + y = 110°$ $S\hat{B}D = 110°$ $\therefore$ SD not diameter [line does not subtend 90° $\angle$] *SD nie 'n middellyn [lyn onderspan nie 90°$\angle$]* **OR/*OF*** $A\hat{S}T = \hat{C} + \hat{D}_2$ [ext $\angle$ of/*buite* $\angle$ ΔSCD] $\hat{C} = 100° - 30° = 70°$ $S\hat{B}D = 180° - 70°$ [opp $\angle$s cyclic quad/ teenoorst $\angle$e kdvh] $= 110°$ $\therefore$ SD not diameter [line does not subtend 90° $\angle$] *SD nie 'n middellyn [lyn onderspan nie 90°$\angle$]*	✓ S ✓ S ✓ S ✓ R (4) ✓ S ✓ S ✓ S ✓ R (4)
		[16]

QUESTION/*VRAAG* 10



10.1.1	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s = *kde onderspan* =∠*e*] $\hat{D}_2 = x$ [exterior angle of cyclic quad/*buite∠ koordevh.*] $\therefore \hat{C}_2 = 90° - x$ [sum of ∠s of/*som v ∠e,* ΔDCM] $\therefore \hat{C}_2 = \hat{A}_1 = 90° - x$ ∴ MC is a tangent to the circle at C [converse: tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]* **OR/*OF*** $\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan* =∠*e*] $\hat{C}_1 + \hat{C}_2 = x$ [sum of ∠s of/*som v ∠e,* ΔACM] $\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$ ∴ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]* **OR/*OF*** In ΔAMC and ΔACB: $\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan* =∠*e*] $A\hat{M}C = A\hat{C}B = 90°$ [given] $\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$	✓ S ✓R ✓ S/R ✓ $\hat{C}_2 = 90° - x$ ✓ R (5) ✓ S ✓ R ✓✓ $\hat{C}_1 + \hat{C}_2 = x$ ✓ R (5) ✓ S ✓ R ✓✓ $\hat{C}_1 + \hat{C}_2 = x$

	$\therefore$ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad.] *[bewys **OF** buite $\angle$ v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of $\angle$s in Δ] [*Bewys **OF** som v $\angle$e in Δ]* ΔACB ||| ΔCMD [$\angle, \angle, \angle$] **OR/*OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad.] *[bewys **OF** buite $\angle$ v koordevh]* $A\hat{C}B = A\hat{M}C = 90°$ [given/*gegee*] ΔACB ||| ΔCMD [$\angle, \angle, \angle$] **OR/*OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad] *[bewys **OF** buite $\angle$ v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of $\angle$s in Δ] *[Bewys **OF** som v $\angle$e in Δ]* $A\hat{C}B = A\hat{M}C = 90°$ [given **OR** sum of $\angle$s in Δ] *[gegee **OF** som v $\angle$e in Δ]* ΔACB ||| ΔCMD	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.1	$\frac{BC}{MD} = \frac{AB}{DC}$ [ΔACB ||| ΔCMD] $\frac{DC}{MD} = \frac{AB}{DC}$ [BC = DC] $\therefore DC^2 = AB \times MD$ In ΔAMC and/*en* ΔCMD $\hat{M}$ is common/*gemeen* $\hat{A}_1 = \hat{C}_2$ [tan chord th /*raaklyn koordst*] **OR/*OF*** $\hat{C}_1 + \hat{C}_2 = \hat{B} = \hat{D} = x$ [tan chord th /*raaklyn koordst* **OR/*OF*** exterior $\angle$ of cyclic quad/ *buite $\angle$ v kdvh*] ΔAMC ||| ΔCMD [$\angle, \angle, \angle$] $\frac{AM}{CM} = \frac{CM}{MD}$ $\therefore CM^2 = AM \times MD$ $\therefore \frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$ $= \frac{AM}{AB}$	✓ $\frac{BC}{MD} = \frac{AB}{DC}$ ✓ $DC^2 = AB \times MD$ ✓ S ✓ S ✓ $CM^2 = AM \times MD$ ✓ $\frac{AM \times MD}{AB \times MD}$ (6)

	OR/*OF* $\frac{AC}{MC}=\frac{AB}{DC}$ [ΔACB ||| ΔCMD] $\therefore CM\times AB = AC\times DC$ In ΔAMC and/*en* ΔACB $\hat{C}=\hat{M}=90°$ [given] $\hat{A}_1=\hat{A}_2$ [proven] **OR/*OF*** $A\hat{C}M=\hat{B}=x$ [proven] ΔAMC ||| ΔACB [$\angle, \angle, \angle$] $\frac{AC}{AM}=\frac{BC}{MC}$ $\therefore AC\times MC = AM\times BC$ $\therefore AC=\frac{BC.AM}{MC}$ $CM\times AB=\frac{BC.AM}{MC}\times DC$ $CM^2=\frac{DC.AM}{AB}\times DC$ [BC = DC] $\frac{CM^2}{DC^2}=\frac{AM}{AB}$	✓ $\frac{AC}{MC}=\frac{AB}{DC}$ ✓ S ✓ S ✓ $AC.MC = AM.BC$ ✓ equating ✓ S (6)
10.2.2	In ΔDMC: $\frac{CM}{DC}=\sin x$ $\frac{CM^2}{DC^2}=\sin^2 x$ $\frac{AC}{AB}=\frac{CM}{DC}$ $\therefore\frac{AM}{AB}=\sin^2 x$ **OR/*OF*** In ΔABC: $\sin x=\frac{AC}{AB}$ In ΔAMC: $\sin x=\frac{AM}{AC}$ $\sin x.\sin x=\frac{AC}{AB}\times\frac{AM}{AC}=\frac{AM}{AB}$	✓ trig ratio ✓ square both sides (2) ✓ 2 equations for $\sin x$ ✓ product (2)
		[16]

TOTAL/*TOTAAL*: 150